Exercise 13

Calculate y'.

$$y = \frac{e^{1/x}}{x^2}$$

Solution

Calculate y' by using the chain and quotient rules.

$$\begin{aligned}
y' &= \frac{d}{dx}\left(\frac{e^{1/x}}{x^2}\right) \\
&= \frac{\left[\dfrac{d}{dx}(e^{1/x})\right]x^2 - \left[\dfrac{d}{dx}(x^2)\right]e^{1/x}}{(x^2)^2} \\
&= \frac{\left[(e^{1/x})\cdot\dfrac{d}{dx}\left(\dfrac{1}{x}\right)\right]x^2 - (2x)e^{1/x}}{x^4} \\
&= \frac{\left[(e^{1/x})\cdot\left(-\dfrac{1}{x^2}\right)\right]x^2 - 2xe^{1/x}}{x^4} \\
&= \frac{-e^{1/x} - 2xe^{1/x}}{x^4} \\
&= \frac{-e^{1/x}(1+2x)}{x^4}
\end{aligned}$$